

# Effect of proton-proton Coulomb repulsion on soft dipole excitations of light proton-rich nuclei

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We perform three-body model calculations for soft dipole excitations of the proton-rich Borromean nucleus  $^{17}\text{Ne}$ . To this end, we assume that  $^{17}\text{Ne}$  takes the  $^{15}\text{O}+p+p$  structure, in which the two valence protons are excited from the  $0^+$  ground state configuration to  $1^-$  continuum states. We employ a density-dependent contact force for the nuclear part of the pairing interaction, and discretize the continuum states with the box boundary condition. We show by explicitly including the Coulomb interaction between the valence protons that the Coulomb repulsion does not significantly alter the E1 strength distribution. We point out that the effect of the Coulomb repulsion in fact can be well simulated by renormalizing the nuclear pairing interaction.

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Properties of unstable nuclei with large excess of proton or neutron are one of the most important current topics of nuclear physics. In several radioactive beam facilities in the world, many unstable nuclei far from the  $\beta$ -stability line have been discovered[1–4]. In particular, neutron-rich unstable nuclei have been extensively studied and some exotic features have been observed. This includes a large concentration of the dipole strength distribution at low energies[5, 6], referred to as a “soft dipole excitation”. This type of excitation is naively understood as an oscillation between weakly bound valence nucleon(s) and the core nucleus[7, 8]. The relation between the soft dipole excitation and a largely extended density distribution, that is, a halo or skin property has been discussed for light neutron-rich nuclei, both experimentally [5, 6, 9–14] and theoretically [15–20]. Recently, the neutron halo structure has been discussed also in the  $^{31}\text{Ne}$  nucleus, based on the measured large Coulomb break-up cross sections[21].

In contrast to neutron-rich nuclei, proton-rich nuclei have been less studied. It has not been fully clarified whether similar exotic features are present also in proton-rich nuclei. For instance, a proton halo structure in *e.g.*,  $^8\text{B}$ ,  $^{12}\text{N}$ ,  $^{17}\text{F}$ , and  $^{17}\text{Ne}$  has been discussed[8, 22–24], but no clear evidence has been obtained so far.

In order to investigate proton-rich unstable nuclei and discuss their similarities and differences to neutron-rich nuclei, it is indispensable to assess the effect of the Coulomb repulsion between valence protons. In the previous work, we analyzed the ground state properties of  $^{17}\text{Ne}$  using a three-body model [25]. We have shown that the effect of the Coulomb repulsion is weak enough and the two valence protons in the ground state of  $^{17}\text{Ne}$  have a spatially compact configuration, that is, the diproton correlation, similar to a dineutron correlation in neutron-rich Borromean nuclei [26–30]. We have also shown that the effect of the Coulomb interaction between the valence protons can be well accounted for by renormalizing the nuclear interaction. A similar conclusion was achieved recently also by Nakada and Yamagami, who

performed Hartree-Fock-Bogoliubov (HFB) calculations for  $N=20, 28, 50, 82$ , and 126 isotones [31].

In this paper, as a continuation of the previous study, we discuss the effect of the Coulomb repulsion on excited states of  $^{17}\text{Ne}$ . Our interest is to investigate whether a similar renormalization for the Coulomb repulsion works also for the soft dipole excitation, which plays an important role in the astrophysical two-proton capture on  $^{15}\text{O}$ [24].

We assume the  $^{17}\text{Ne}$  nucleus as a three-body system composed of an inert spherical core nucleus  $^{15}\text{O}$  and two valence protons. The three-body Hamiltonian in the three-body rest frame reads

$$H = h^{(1)} + h^{(2)} + \frac{\mathbf{p}_1 \cdot \mathbf{p}_2}{A_C m} + V_{pp}(\mathbf{r}_1, \mathbf{r}_2), \quad (1)$$

$$h^{(i)} = \frac{\mathbf{p}_i^2}{2\mu} + V_{pC}(\mathbf{r}_i), \quad (2)$$

where  $m$  and  $A_C$  are the nucleon mass and the mass number of the core nucleus, respectively.  $h^{(i)}$  is the single-particle (s.p.) Hamiltonian for a valence proton, in which  $\mu = mA_C/(A_C + 1)$  is the reduced mass and  $V_{pC}$  is the potential between the proton and the core nucleus. The third term in Eq.(1) is a two-body part of the recoil kinetic energy of the core nucleus. For the proton-core potential  $V_{pC}$ , we employ a Woods-Saxon (WS) plus Coulomb potential in the same manner as in the previous work [25]. We use the same parameters for the WS potential as in those listed in Ref.[25].

We solve the three-body Hamiltonian, Eq. (1), by expanding the wave function on the uncorrelated basis as,

$$\Psi^{(J,M)}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{k_1 \leq k_2} \alpha_{k_1, k_2} \tilde{\psi}_{k_1, k_2}^{(J,M)}(\mathbf{r}_1, \mathbf{r}_2), \quad (3)$$

where

$$\begin{aligned} \tilde{\psi}_{k_1, k_2}^{(J, M)}(\mathbf{r}_1, \mathbf{r}_2) &= \frac{1}{\sqrt{2}} [1 - (-)^{j_1 + j_2 - J} \delta_{k_1, k_2}]^{-1} \\ &\times \sum_{m_1, m_2} \langle j_1 m_1 j_2 m_2 | JM \rangle \\ &\times [\phi_{k_1, m_1}(\mathbf{r}_1) \phi_{k_2, m_2}(\mathbf{r}_2) \\ &- \phi_{k_2, m_2}(\mathbf{r}_1) \phi_{k_1, m_1}(\mathbf{r}_2)]. \end{aligned} \quad (4)$$

Here,  $\phi_{km}(\mathbf{r})$  is a s.p. wave function with  $k = (n, l, j)$ , while  $J$  and  $M$  are the total angular momentum of the two-proton subsystem and its  $z$  component, respectively.  $\alpha_{k_1, k_2}$  is the expansion coefficient. The summation in Eq. (3) is restricted to those combinations which satisfy  $\pi = (-)^{l_1 + l_2}$  for a state with parity  $\pi$ .

In the actual calculations shown below, we include the s.p. angular momentum  $l$  up to 5. We have confirmed that our results do not change significantly even if we include up to a larger value of  $l$ . In order to take into account the effect of the Pauli principle, we explicitly exclude the  $1s_{1/2}$ ,  $1p_{3/2}$ , and  $1p_{1/2}$  states from Eq. (3), which are occupied by the protons in the core nucleus.

For the pairing interaction  $V_{pp}$ , we assume a density-dependent contact interaction [17, 19, 26] together with the Coulomb repulsion,  $V_{pp} = V_{pp}^{(N)} + V_{pp}^{(C)}$ , as in the previous work [25]. We take a cutoff energy  $E_{\text{cut}} = 30$  MeV and include those configurations which satisfy

$$\epsilon_{n_1 l_1 j_1} + \epsilon_{n_2 l_2 j_2} \leq \frac{A_C + 1}{A_C} E_{\text{cut}}, \quad (5)$$

where  $\epsilon_{nlj}$  is a s.p. energy [19]. Within this truncated space, we determine the strength of the nuclear part of the pairing interaction,  $V_{pp}^{(N)}$ , using the empirical value of the neutron-neutron scattering length,  $a_{nn} = -18.5$  fm [19]. The parameters for the density dependence are adjusted so as to reproduce the experimental value of the two-proton separation energy of  $^{17}\text{Ne}$ ,  $S_{2p} = 0.944$  MeV.

In our calculations, the continuum s.p. spectra are discretized within a box of  $R_{\text{box}} = 30$  fm. Thus, energies of the two-proton  $1^-$  states as well as the E1 strength distribution are also discretized. The E1 strength function from the ground state is defined as

$$S(E) = \sum_i \mu_i \delta(E - \hbar\omega_i) \quad (6)$$

where  $\hbar\omega_i = E_i - E_{\text{g.s.}}$ ,  $E_{\text{g.s.}}$  being the energy of the ground state, and  $\mu_i$  is the  $B(\text{E1})$  strength for  $i$ -th  $1^-$  two-proton state,

$$\mu_i = 3 \left| \left\langle \Psi_i^{(1,0)} \mid \hat{D}_0 \mid \Psi_{\text{g.s.}} \right\rangle \right|^2, \quad (7)$$

with the E1 operator given by

$$\hat{D}_\mu = e \left( \frac{A_C - Z_C}{A_C + 2} \right) [r_1 Y_{1\mu}(\hat{\mathbf{r}}_1) + r_2 Y_{1\mu}(\hat{\mathbf{r}}_2)]. \quad (8)$$

Using the strength function  $S(E)$ , we can also calculate the  $k$ -th moment of energy defined as

$$S_k = \int dE E^k S(E) = \sum_i (\hbar\omega_i)^k \mu_i. \quad (9)$$

Notice that  $S_0$  and  $S_1$  correspond to the direct and energy-weighted sum of  $dB(\text{E1})/dE$ , respectively.

From the completeness of the  $1^-$  basis, we can estimate the sum-rule-values as

$$S_{0,\text{SR}} = \frac{3}{\pi} e^2 \left( \frac{A_C - Z_C}{A_C + 2} \right)^2 \langle \Psi_{\text{g.s.}} | r_{2N-C}^2 | \Psi_{\text{g.s.}} \rangle, \quad (10)$$

$$S_{1,\text{SR}} = \frac{9}{4\pi} e^2 \left( \frac{A_C - Z_C}{A_C + 2} \right)^2 \frac{A_C + 2}{A_C m} \hbar^2, \quad (11)$$

where  $r_{2N-C} = (\mathbf{r}_1 + \mathbf{r}_2)/2$  is the distance between the center of mass of the two valence protons and the core nucleus. For the core nucleus  $^{15}\text{O}$  ( $A_C = 15$  and  $Z_C = 8$ ), we obtain  $S_{0,\text{SR}} = 1.49 e^2 \text{fm}^2$  and  $S_{1,\text{SR}} = 5.69 e^2 \text{fm}^2 \text{MeV}$ . Due to the Pauli forbidden transitions, the actual value of  $S_0$  is smaller than  $S_{0,\text{SR}}$ , while  $S_1$  is larger than  $S_{1,\text{SR}}$ .

TABLE I: The results for the soft dipole excitations of  $^{17}\text{Ne}$  obtained with the three-body model of  $^{15}\text{O} + p + p$ .  $S_0$  and  $S_1$  are the non-energy weighted sum rule and the energy weighted sum rule, respectively.  $E_{\text{cent}} = S_1/S_0$  is the centroid energy of the dipole strength distribution.  $\delta E_{\text{cent}}$  is a shift of the centroid energy with respect to the result of the exact treatment of the Coulomb interaction.

pairing	$S_0$	$S_1$	$E_{\text{cent}}$	$\delta E_{\text{cent}}$
	( $e^2 \text{fm}^2$ )	( $e^2 \text{fm}^2 \text{MeV}$ )	(MeV)	(MeV)
Nucl. + Coul.	1.206	11.02	9.140	0
Nucl. only	1.206	10.45	8.666	-0.47
Ren. Nucl.	1.205	10.86	9.017	-0.12
No pairing	1.206	15.50	12.86	3.72

Our main results are summarized in Fig. 1 and Table I. For a plotting purpose, we smear the Dirac delta function in the E1 distribution  $S(E)$  with a Cauchy-Lorentz function

$$\frac{dB(\text{E1})}{dE} = \sum_i \mu_i \frac{\Gamma}{\pi} \frac{1}{(E - \hbar\omega_i)^2 + \Gamma^2} \quad (12)$$

with the width parameter  $\Gamma$  of 1.0 MeV. In order to discuss the effect of the Coulomb part of the pairing interaction, we also perform the calculations with two other treatments for the Coulomb interaction. One is to switch off the Coulomb interaction (“Nucl. only”), that is,  $V_{pp} = V_{pp}^{(N)}$ , keeping the same values for the parameters of the nuclear pairing interaction  $V_{pp}^{(N)}$  as in the full calculation (“Nucl.+Coul.”). The other is again to use the nuclear interaction only, but renormalize the parameters, that is,  $V_{pp} = \tilde{V}_{pp}^{(N)}$  (“Ren. Nucl.”). To renormalize

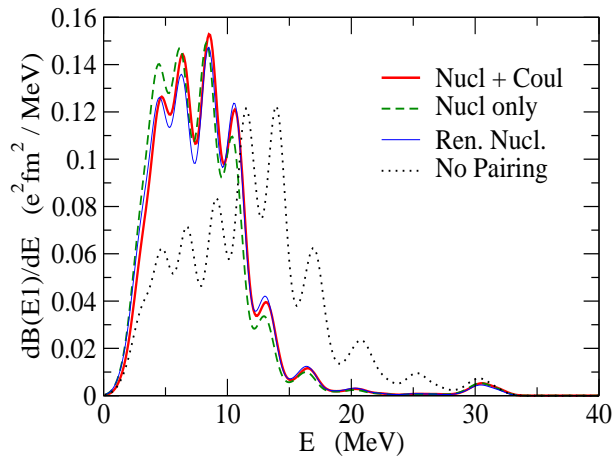


FIG. 1: (Color online) Comparison of the E1 strength distributions for  $^{17}\text{Ne}$  obtained with several treatments for the Coulomb interaction between the valence protons. The solid line is obtained by fully including the Coulomb interaction, while the dashed line is obtained by switching off the Coulomb repulsion. The thin solid line is the result of the renormalized nuclear interaction, which is readjusted to reproduce the ground state energy without the Coulomb interaction. The dotted line denotes the result without any pairing interaction. These distributions are smeared with the Cauchy-Lorentz function, Eq. (12), with  $\Gamma = 1.0$  MeV.

the interaction, we use the empirical proton-proton scattering length,  $a_{pp} = -7.81$  fm, instead of the neutron-neutron scattering length  $a_{nn}$ , and determine the other parameters so that the two-proton separation energy  $S_{2p}$  is reproduced. This leads to about 10.0% reduction of the strength of the pairing interaction. Notice that this value for the reduction factor is consistent with the finding of Ref. [31]. See Ref.[25] for further details of the procedure. For a comparison, we also show the results without any pairing interaction. These treatments for the Coulomb interaction are applied only to the excited states, while the same ground state wave function is used for all the cases. That is, the ground state is calculated with the full treatment of the Coulomb interaction, that yields 76% of  $(d_{5/2})^2$  and 16% of  $(s_{1/2})^2$ . This ground state is used also for the no-pairing calculation for the dipole excitations. (These values for the occupation probabilities are slightly different from those in Ref.[25], as we use a smaller  $E_{\text{cut}}$  in this paper. We have confirmed that the dipole strength distribution does not change much even though we use the smaller value of  $E_{\text{cut}}$ .) Notice that our results for  $S_0$  are consistent with the result of Grigorenko *et al* [24], that is,  $S_0 = 1.56 e^2\text{fm}^2$  for  $(s_{1/2})^2=48\%$  and

$S_0 = 1.07 e^2\text{fm}^2$  for  $(s_{1/2})^2=5\%$ . Table I also lists the centroid energy defined as  $E_{\text{cent}} \equiv S_1/S_0$  [32], and its relative value with respect to the result of the full treatment of the Coulomb interaction.

From Fig. 1 and Table I, we can see that the pairing interaction shifts considerably the E1 strength distribution towards the low energy region, similarly to the dipole distribution in neutron-rich nuclei [6, 20]. This large shift of the E1 strength distribution originates mainly from the nuclear part of the pairing interaction. If the Coulomb part is switched off, the strength distribution is shifted only slightly, as is shown in Fig. 1 by the dashed line. The shift of the centroid energy,  $\delta E_{\text{cent}}$ , is only  $-0.47\text{MeV}$ . The sign and the magnitude of  $\delta E_{\text{cent}}$  is consistent with the result of Hartree-Fock+RPA calculations for medium-heavy nuclei shown in Ref. [32], although  $\delta E_{\text{cent}}$  for the soft dipole excitation in  $^{17}\text{Ne}$  is somewhat larger.

The result with the renormalized nuclear pairing interaction is shown by the thin solid line in Fig. 1. As one can see, the result of the full calculation (the thick solid line) is well reproduced by this prescription. We can thus conclude that the renormalization works well not only for the ground state [25, 31], but also for the dipole excitations.

In summary, we discussed the influence of the Coulomb repulsion between the valence protons upon the soft dipole excitation in  $^{17}\text{Ne}$ . We showed that the effect of the Coulomb repulsion is so weak that the main feature of the dipole response is similar between neutron-rich and proton-rich weakly bound nuclei. We also showed that the effect of the Coulomb interaction can be well mocked up by renormalizing the nuclear pairing interaction. This renormalization works both for the ground state and for the dipole excitations, and thus from a practical point of view one can use a renormalized pairing interaction in order to understand the structure of proton-rich nuclei.

One of the current topics of proton-rich nuclei is two-proton radioactivity. Bertulani, Hussein, and Verde argued that the final state interaction plays an important role in discussing the energy and the angular correlations in a two-proton emission process [33]. It would be an interesting future work to investigate how the renormalization works for those correlations. A work towards this direction is now in progress.

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